Technical Notes

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Direct Approach to the Analysis of Control Reversal and its Sensitivity

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Nomenclature

$[A]$ = aerodynamic influence coefficients matrix
$[F]$ = structural flexibility matrix
$[K]$ = structural stiffness matrix
$\{M_{pe}\}$ = vector of aerodynamic roll moments due to unit $q\{\Delta P_e\}$
$pb/2V$ = nondimensional roll rate
q = dynamic pressure
$q\{\Delta P_e\}$ = aeroelastic load variation
$q\{\Delta P_p\}$ = aerodynamic load variation due to unit $pb/2V$
$q\{\Delta P_\delta\}$ = aerodynamic load variation due to unit aileron deflection
qM_{pp} = aerodynamic roll moment for unit $pb/2V$
$qM_{p\delta}$ = aerodynamic roll moment due to unit aileron deflection
$q\{H_{pu}\}$ = vector of generalized aerodynamic roll moments due to elastic deformations
$q\{H_{up}\}$ = vector of generalized aerodynamic loads to unit $pb/2V$
$q\{H_{u\delta}\}$ = vector of generalized aerodynamic load due to unit aileron deflection
$q[H_{uu}]$ = aerodynamic stiffness matrix
δ = aileron deflection

Introduction

THE loss of control efficiency at increasing dynamic pressure is a major concern in the design of highly performing flexible aircraft. A straightforward and simple way to guarantee the maintenance of an acceptable control efficiency is to require a high reversal dynamic pressure, i.e., well beyond the maximum achievable at any allowed flight condition. This type of specification has often to be taken into account during the structural optimization for minimum weight either by mathematical programming or by optimality criteria. The usual method of evaluating control reversal is to follow the steady response rate at different flight conditions and to interpolate for its null value. This approach is useful for aeroelastic analyses since an extensive evaluation of controls efficiency at different flight conditions is often required but makes it awkward to include the control reversal constraint in the structural optimization because of the difficulty of calculating its sensitivity to variations of structural design variables. This brief note shows that control reversal can be cast into a simple eigenvalue problem whose solution and sensitivity calculations can be obtained by well known formulas requiring standard numerical methods.

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Control Reversal

For the sake of simplicity, we will refer to the aileron reversal problem. Basic texts in aeroelasticity, e.g., Refs. 1-2, usually present the problem of steady-state response to a step aileron command in a flexibility formulation (see the nomenclature for a list of symbols not explicitly explained)

$$\begin{bmatrix} -qM_{pp} & -q\{M_{pe}\}^T \\ q[F]\{\Delta P_p\} & q[F]-[A] \end{bmatrix} \begin{Bmatrix} pb/2V \\ \{\Delta P_e\} \end{Bmatrix} = q \begin{Bmatrix} M_{p\delta} \\ -[F]\{\Delta P_\delta\} \end{Bmatrix} \delta \tag{1a}$$

A more rarely used approach is the adoption of a static modal formulation, i.e., the use of appropriate displacement functions to cast the problem in a stiffness form, as it is usually done in flutter analyses,[3] i.e.,

$$\begin{bmatrix} -qM_{pp} & -q\{H_{pu}\}^T \\ -q\{H_{up}\} & [K]-q[H_{uu}] \end{bmatrix} \begin{Bmatrix} pb/2V \\ \{u\} \end{Bmatrix} = q \begin{Bmatrix} M_{p\delta} \\ \{H_{u\delta}\} \end{Bmatrix} \delta \tag{1b}$$

We will not digress here about the advantages of either approaches. Instead we note that in any case the response rate is obtained by solving Eqs. (1) for $pb/2V$ for a unit δ at varying Mach number and dynamic pressure q provided that the coefficient matrix is not singular, i.e., excluding a dynamic divergence condition for Eqs. (1). The inversion condition can then be simply defined by imposing $pb/2V=0$ for whatever δ. This leads to the following eigenvalue problems

$$\begin{bmatrix} 0 & 0 \\ 0 & [A] \end{bmatrix} \begin{Bmatrix} \delta \\ \{\Delta P_e\} \end{Bmatrix} = q \begin{bmatrix} M_{p\delta} & \{M_{pe}\}^T \\ [F]\{\Delta P_\delta\} & [F] \end{bmatrix} \begin{Bmatrix} \delta \\ \{\Delta P_e\} \end{Bmatrix} \tag{2a}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & [K] \end{bmatrix} \begin{Bmatrix} \delta \\ \{u\} \end{Bmatrix} = q \begin{bmatrix} M_{p\delta} & \{H_{pu}\}^T \\ \{H_{u\delta}\} & [H_{uu}] \end{bmatrix} \begin{Bmatrix} \delta \\ \{u\} \end{Bmatrix} \tag{2b}$$

which must be solved for the minimum positive q different from zero to exclude the trivial inversion condition $q=0$, which is clearly a solution of Eqs. (2). Since in Eqs. (2) the first line of the left-hand side is null, we can obtain δ from the right-hand side of the same line. After substitution into the remaining equations, we have

$$[A]\{\Delta P_e\} = q[F]\left([I] - \frac{1}{M_{p\delta}}\{\Delta P_\delta\}\{M_{pe}\}^T\right)\{\Delta P_e\} \tag{3a}$$

$$[K]\{u\} = q\left([H_{uu}] - \frac{1}{M_{p\delta}}\{H_{u\delta}\}\{H_{pu}\}^T\right)\{u\} \tag{3b}$$

We note that Eq. (3a) can be transformed into the same form of Eq. (3b). In fact if we call

$$[A]\{\Delta P_e\} = \{v\} \tag{4}$$

then Eq. (3a) can be written

$$[F]^{-1}\{v\} = q\left([I] - \frac{1}{M_{p\delta}}\{\Delta P_\delta\}\{M_{pe}\}^T\right)[A]^{-1}\{v\} \quad (5)$$

Both Eq. (3a) and Eq. (5) can be put in the form

$$[K_s]\{x\} = q[K_a]\{x\} \quad (6)$$

where $[K_s]$ is a symmetric and positive definite structural stiffness matrix; while $[K_a]$ is a generally nonsymmetric aerodynamic stiffness matrix. Generally, since the lowest eigenvalue of Eq. (6) is well separated from the others, the inverse power method can be used to obtain the inversion dynamic pressure by iterating the following

$$\{x\}_{i+1} = [K_s]^{-1}[K_a]\{x\}_i \quad (7a)$$

$$\{y\}_{i+1} = [K_s]^{-1}[K_a]^T\{y\}_i \quad (7b)$$

Once the right and left eigenvectors, $\{x\}$ and $\{y\}$, have converged we calculate q_i by

$$q_i = \frac{\{y\}^T[K_s]\{x\}}{\{y\}^T[K_a]\{x\}} \quad (7c)$$

So if the constraint is expressed as

$$1/q_i \le 1/q_{i\,(\text{acceptable})} \quad (8)$$

Eq. (7c) shows that a design can be forced to satisfy Eq. (8) by a uniform scaling of the structural stiffness matrix. If $[K_s]$ is related linearly to the design variables, i.e.,

$$[K_s] = \Sigma t_i [K_{t_i}] \quad (9)$$

this corresponds to a uniform scaling of the design variables and makes it easy to produce a feasible starting point for any optimal design procedure. Assuming that the design variables do not influence the external shape of the aircraft, i.e., $[K_a]_{/t_i} = 0$, the sensitivity to a design variable t_i is given by

$$(1/q_i)_{/t_i} = -\frac{1}{q_i}\frac{\{y\}^T[K_s]_{/t_i}\{x\}}{\{y\}^T[K_s]\{x\}} \quad (10)$$

It can be seen that if we define a generalized displacement vector $\{w\}$ by

$$\{w\} = -\frac{1}{q_i}\frac{\{y\}}{\{y\}^T[K_s]\{x\}} \quad (11)$$

this can be taken as the deformation of the structure when it is loaded by

$$\{P\} = -\frac{1}{\{y\}^T[K_s]\{x\}}[K_a]^T\{y\} \quad (12)$$

In fact since $\{y\}$ satisfies

$$[K_s]\{y\} = q_i[K_a]^T\{y\} \quad (13)$$

by combining Eqs. (11-13) we have

$$[K_s]\{w\} = \{P\} \quad (14)$$

Then Eq. (10) can be rewritten as

$$(1/q_i)_{/t_i} = \{w\}^T[K_s]_{/t_i}\{x\} \quad (15)$$

In this way the derivative of $1/q_i$ is related to an equivalent deformation energy and has the same appearance of the derivatives of a static displacement with $\{w\}$ taking the part of a displacement corresponding to an appropriate dummy unit load.[4] If $[K_s]$ is given by Eq. (9), then Eq. (15) can be rewritten either as

$$(1/q_i)_{/t_i} = \{w\}^T[K_{t_i}]\{x\} \quad (16a)$$

or as

$$(1/q_i)_{/t_i} = \frac{\{w\}^T t_i[K_{t_i}]\{x\}}{t_i} = \frac{C_i}{t_i} \quad (16b)$$

The design variable t_i appears at the denominator in Eq. (16b), and the derivatives can be assumed almost linear in the inverse of the design variable t_i if C_i is approximately constant. Eqs. (16) allow a ready adoption of the formulation just presented either in an optimality criterion or in a convex approximation to be used in a mathematical optimization. It must be noted that C_i is not always positive implying that a stiffening of a particular structural component is not necessarily in favor of a better reversal condition and then it must be excluded from the active set in an optimality procedure, or a direct variable approximation must be used for a convex approximation.[5-6] It must then be noted that this note has contributed a general formulation of controls inversion in the framework of an eigenvalue problem, i.e., Eq. (6), that is formally the same as that of aeroelastic divergence allowing a unification of the techniques used in the solution and sensitivity analysis of both problems. A similar approach has in fact been already adopted to treat the constraint on aeroelastic divergence in optimality design methods.[7]

Concluding Remarks

This Note has shown that the analysis of controls reversal can be formulated as an eigenvalue problem and that its sensitivity can be obtained in a simple way closely resembling that used in calculating the sensitivities of generalized static displacements. Thus the reversal constraint can be included in any optimal structural design process since a feasible design can be easily obtained by uniformly scaling the design variables upward. This inclusion is further eased if the design procedure can already treat the aeroelastic divergence as what has been previously presented unifies the formulation of the two problems.

References

[1]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity,* Addison-Wesley Publishing Co., 1955

[2]Bisplinghoff, R. L., and Ashley, H., *Principles of Aeroelasticity,* Dover, New York, 1962.

[3]Franchi, C., "Un Moderno Inquadramento dei Problemi di Aeroelasticá Statica," Thesis in Aeronautical Engineering, Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Italy, 1988.

[4]Fleury, C., and Gerardin, M., "Optimality Criteria and Mathematical Programming in Structural Weight Optimization," *Computer and Structures,* Vol. 8, Feb. 1978, pp. 7-17.

[5]Starnes, J. H., Jr., and Haftka, R. T., "Preliminary Design of Composite Wings for Buckling, Strength and Displacement Constraints," *Journal of Aircraft,* Vol. 16, No. 8, 1979, pp. 564-570.

[6]Fleury, C., "Structural Weight Optimization by Dual Methods of Convex Programming," *International Journal of Numerical Methods in Engineering,* Vol. 14, No. 12, 1979, pp. 1761-1783.

[7]Taig, I. C., and Kerr, R. I., "Optimization of Aircraft Structures with Multiple Stiffness Requirements," Second Symposium on Structural Optimization, AGARD-CP-123:16, Milan, Italy, April 1973.